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Engineering Notes

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Singularities of Nonlinear Attitude Control with Momentum Management

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Introduction

RECENT efforts in spacecraft control have focused on using nonlinear feedback linearization to develop analytic control laws that perform both spacecraft attitude control and momentum management.[1–5] A new nonlinear control law expressed in terms of the spacecraft attitude, angular rates, and total stored control moment gyroscope momentum[5] is the focus of this Note. The design process makes no small angle, small rates, or negligible cross product of inertia approximations. Through development of the nonlinear control law, a nonlinear transformation that takes the spacecraft dynamic system to a companion form was introduced. The nonlinear transformation is not global and is known to generate singularities in the control law. An expression that defines all of the singularities in the control law has been reported[5] and is given in terms of the spacecraft inertia matrix and the attitude of the spacecraft. In this Note, the behavior of the singularity expression is investigated. A more thorough analysis of the singularities has been presented.[6] Simulation results show the behavior of the nonlinear control law as the singularities are neared.

Control Law Singularities

An important step in feedback linearization involves transforming the nonlinear system into a normal form. The nonlinear transformation is not a global transformation and is valid only in a region around a particular equilibrium point. In general, the associated nonlinear control law has the form

$$\boldsymbol{u} = \boldsymbol{A}(\boldsymbol{x})^{-1}[-\boldsymbol{b}(\boldsymbol{x}) + \boldsymbol{v}] \tag{1}$$

where $\boldsymbol{A}(\boldsymbol{x})$ and $\boldsymbol{b}(\boldsymbol{x})$ depend on the system dynamics and $\boldsymbol{v}$ is chosen to obtain the desired linear behavior of the transformed coordinates. The regions where the nonlinear control law is valid are represented by all of the points where the matrix $\boldsymbol{A}(\boldsymbol{x})$ is nonsingular. For our nonlinear control law,[5] we have $\boldsymbol{A}(\boldsymbol{x}) = -3\eta^3 \boldsymbol{S}_1^L(\boldsymbol{I}_B^L)^{-1}$, and its inverse is given by

$$\boldsymbol{A}^{-1}(\boldsymbol{x}) = -(1/3\eta^3)\boldsymbol{I}_B^L\left(\boldsymbol{S}_1^L\right)^{-1} \tag{2}$$

where

$$\boldsymbol{S}_1^L = \begin{bmatrix} I_{B_{22}}^L - I_{B_{33}}^L & -I_{B_{12}}^L & I_{B_{13}}^L \\ I_{B_{12}}^L & I_{B_{33}}^L - I_{B_{11}}^L & -I_{B_{23}}^L \\ -I_{B_{13}}^L & I_{B_{23}}^L & I_{B_{11}}^L - I_{B_{22}}^L \end{bmatrix} \tag{3}$$

The terms $I_{B_{ij}}^L$ are the inertia elements written in the spacecraft local-vertical, local-horizontal frame, and η is the orbital rate. Therefore, the invertibility of $\boldsymbol{A}(\boldsymbol{x})$ is dependent on the invertibility of $\boldsymbol{S}_1^L$. For $\boldsymbol{S}_1^L$ to be invertible, Δ must be nonzero, where Δ is the determinant of $\boldsymbol{S}_1^L$ and can be written

$$\Delta = \left(I_{B_1}^P - I_{B_2}^P\right)\left(I_{B_2}^P - I_{B_3}^P\right)\left(I_{B_3}^P - I_{B_1}^P\right)\Lambda(\boldsymbol{T}^{PL}) \tag{4}$$

where Λ is

$$\Lambda(\boldsymbol{T}^{PL}) = 1 + 2T_{11}^{PL}T_{23}^{PL}T_{32}^{PL} + 2T_{22}^{PL}T_{13}^{PL}T_{31}^{PL} + 2T_{33}^{PL}T_{12}^{PL}T_{21}^{PL} \tag{5}$$

and T_{ij}^{PL} is the ijth element of $\boldsymbol{T}^{PL}$, the transformation matrix from the local-vertical, local-horizontal frame to the principal axis frame. The elements of $\boldsymbol{I}_B^P$ are the principal inertias.

For Δ to be nonzero, the following conditions must be true: $I_{B_1}^P \neq I_{B_2}^P$, $I_{B_2}^P \neq I_{B_3}^P$, $I_{B_3}^P \neq I_{B_1}^P$, and $\Lambda \neq 0$. The first three conditions are physical attributes of the spacecraft being analyzed and, as such, limit which spacecraft are candidates for use of the nonlinear control law. The condition that Λ be nonzero imposes constraints on the attitude orientations that the spacecraft can achieve.

Attitude Constraints

Because the control law requires that $\Lambda \neq 0$, an understanding of when and where the equality can occur is important. The singularities in a 2-3-1 principal Euler angle rotation sequence are examined here. The behavior of Λ is examined near the points where we expect the spacecraft to operate. The attitudes are known as torque equilibrium attitudes (TEA). For a 2-3-1 rotation, it follows from Eq. (5) that

$$\Lambda(\boldsymbol{\theta}^*) = \sin 2\theta_1^* \sin 2\theta_2^* \sin\theta_3^*\left(\tfrac{1}{4} - \tfrac{3}{4}\cos 2\theta_3^*\right) + \cos 2\theta_1^* \cos 2\theta_2^* \cos 2\theta_3^* \tag{6}$$

where $\boldsymbol{\theta}^*$ denotes the Euler angles of the principal axes relative to the local-vertical, local-horizontal frame. The possible TEA principal Euler angle orientations are given by

$$\boldsymbol{\theta}^* = (m\pi/2 \quad n\pi/2 \quad p\pi/2)^T \tag{7}$$

where m, n, and p can take any integer values.

Collecting the sets of Euler angles that result in $\Lambda = 0$ generates a family of spacecraft orientations that cause the control law to become singular. This family of orientations leads to a surface of singular points in the θ_1^*, θ_2^*, θ_3^* space. For a 2-3-1 rotation sequence, the singular surface around $\boldsymbol{\theta}^* = (0, 0, 0)$ for a spacecraft with $I_{B_1}^P \neq I_{B_2}^P$, $I_{B_2}^P \neq I_{B_3}^P$, and $I_{B_3}^P \neq I_{B_1}^P$ is shown in Fig. 1. The two surfaces shown form boundaries separating the region around the TEA at $\boldsymbol{\theta}^* = (0, 0, 0)$ from regions around other TEA. If a spacecraft is initially oriented so that the Euler angles are in the region around the TEA at $\boldsymbol{\theta}^* = (0, 0, 0)$, then the orientations that the spacecraft passes through to reach the TEA must remain in that region to avoid crossing the singular surface. Around some of the other TEA, the

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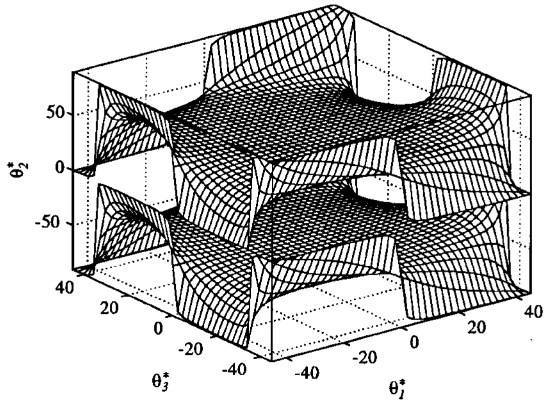


Fig. 1a Singularity surface around $\theta^* = (0, 0, 0)$.

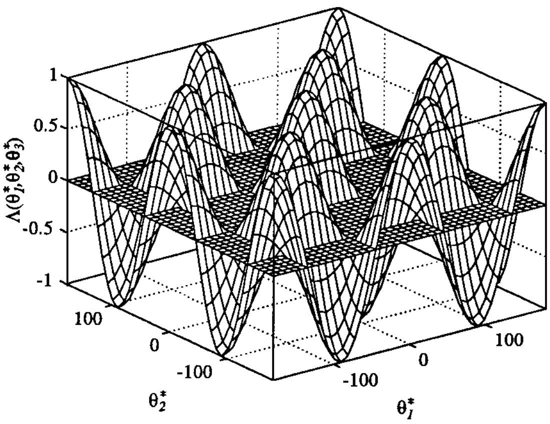
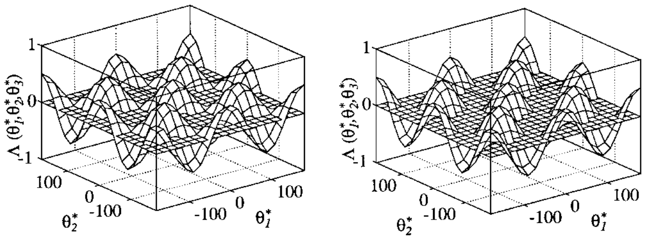
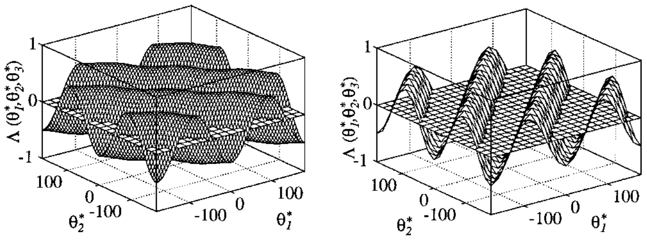


Fig. 1b Behavior of Λ when $\theta_3^* = 0$.



a) $\theta_3^* = 30$ deg **b) $\theta_3^* = -30$ deg**

Fig. 2 Family of Λ plots.



a) $\theta_3^* = 60$ deg **b) $\theta_3^* = -60$ deg**

Fig. 3 Family of Λ plots.

shapes of the singular surfaces are different. Figures 1b, 2, and 3 show the family of plots as θ_3^* varies from -60 to 60 deg. This family of plots shows that the values of Λ are symmetric about the $\theta_1^* = \theta_2^*$ and $-\theta_2^*$ planes in space. The values of Λ range between -1 and 1. Considering the TEA in Eq. (7), we have that $\Lambda(\theta_1^*, \theta_2^*, \theta_3^*) = (-1)^{m+n+p}$. Therefore, at any TEA, the value of Λ will be either -1 or 1, which implies that the TEA are not singular points of the control law. In fact, the TEA are extrema of the function $\Lambda(\theta_1^*, \theta_2^*, \theta_3^*)$. To determine the classification of the extrema, the matrix of second partials is examined. Evaluating the matrix of partial derivatives at the TEA leads to

$$\left.\frac{\partial^2\Lambda}{\partial\boldsymbol{\theta}^{*2}}\right|_{\text{TEA}} = -4(-1)^{m+n}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}$$

if p is even and

$$\left.\frac{\partial^2\Lambda}{\partial\boldsymbol{\theta}^{*2}}\right|_{\text{TEA}} = 4(-1)^{m+n}\begin{bmatrix} 1 & (-1)^{(p-1)/2} & 0 \\ (-1)^{(p-1)/2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

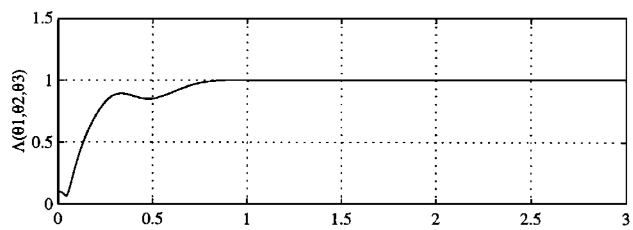
if p is odd. Therefore, if p is even, the three eigenvalues of the second partial derivative matrix are $-4(-1)^{m+n}$, and the TEA extrema are either maxima or minima depending on whether $m + n$ is even or odd. If p is odd, then the eigenvalues are $8(-1)^{m+n}$, $4(-1)^{m+n}$, and 0, and the extrema are inflection points.

From the preceding analysis, it appears that all of the TEA are located as far as they can be from the singular surface. Therefore, operation of the nonlinear control law around the TEA should avoid all of the singular points. However, initial orientations near the TEA do not guarantee that the state trajectories will not cross the singular surface. If, for instance, an initial angular rate is large, it is possible that the controller will be unable to slow the spacecraft angular rate before a singular attitude orientation is reached. The ability of the nonlinear controller to drive the spacecraft to a TEA and avoid the singular surface is heavily dependent on the initial conditions of the system. Based on the known behavior of Λ near its roots, it may be possible to generate a reference input signal that can drive the system without crossing the singular surface.

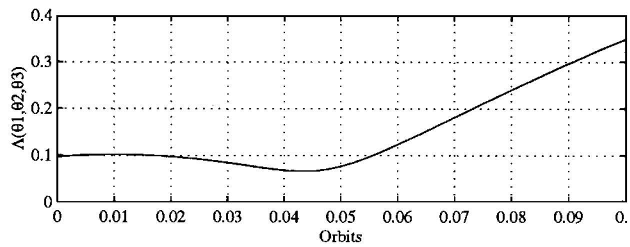
Results

A simulation that highlights the nonlinearities of the problem is shown in this section. In this case, the spacecraft considered is the International Space Station Alpha after Flight 40-12R, with principal inertias $I_{B_1}^P = 7.0626 \times 10^7$, $I_{B_2}^P = 5.5393 \times 10^7$, and $I_{B_3}^P = 1.1521 \times 10^8$. The initial conditions are $\boldsymbol{\theta} = (30, -30, -30)$ deg, $\boldsymbol{\omega}_B^B = (0, -\eta, 0)$, and $\boldsymbol{h}_W^B = (0, 0, 0)$. The attitude control and momentum management will drive the system to the TEA $\boldsymbol{\theta}^* = (0, 0, 0)$.

The values of Λ indicate how close the control law is to becoming singular during the course of the simulation. Figure 4 shows how Λ evolves during the simulation and indicates that the system never crosses the singular surface. Figure 4a shows that Λ starts very close to zero and stabilizes at a value of 1. The value of Λ during the initial system response is shown in Fig. 4b. Between 0.04 and 0.05 orbit, the value of Λ is closest to zero. This is also where the behavior of the control exhibits very nonlinear behavior.



a) Λ



b) Λ ($0 \le t \le 0.1$)

Fig. 4 Value of singular function Λ.

Conclusions

An explicit function associated with the invertibility of the nonlinear attitude controller with momentum management has been obtained. The function is given in terms of the mass properties of the spacecraft and the attitude of principal body axes relative to the local-vertical, local-horizontal frame. The TEA were shown to occur at orientations that are as far as can be from the singularity surface. In fact, the TEA occur at local minima and maxima of the singularity function. Knowledge of the singularity function may provide the basis for development of an explicit method to avoid singularities in the feedback linearized control law.

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Aerodynamic Parameter Estimation for High-Performance Aircraft Using Extended Kalman Filtering

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Introduction

THE design of flight control laws, the verification of performance predictions, and the implementation of flight simulations are all tasks that require a mathematical model for the dynamics of an aircraft. This dynamic model is typically characterized by coefficients or parameters whose numerical values must be determined for various flight conditions of interest. Among the most important of these are the parameters in the mathematical models for the aerodynamic forces and moments, often referred to as the aerodynamic coefficients.[1] Numerical values for these parameters are often first derived from wind-tunnel test data. However, the wind-tunnel conditions typically do not replicate the actual flight environment. It is desirable then to derive estimates for the aerodynamic coefficients directly from flight test data.

A number of parameter estimation methods, such as maximum likelihood[2] (ML) and linear regression,[3] have been applied to derive aerodynamic coefficients from aircraft flight test data. Filter-based methods, such as the extended Kalman filter[4] (EKF), have also been used with varying degrees of success (see bibliography in Ref. 5). One advantage of the EKF approach relative to most other approaches (including most ML formulations and essentially all least squares methods) is that it places no linearity restrictions on the form in which either the states or the parameters appear in the dynamic equations describing the system. It also does not require the parameters to be time invariant, nor does it require stability of the system. Finally, the EKF produces estimates of the parameters that approximately minimize the mean square error in the parameter estimates themselves, as opposed to minimizing a cost function that is based on matching the output variable behavior given a specific input trajectory, which is what most ML and least squares techniques are designed to do.

The EKF approach, however, requires the designer to select the statistical properties of the process and measurement noises and a model for the parameter dynamics. Some of this information is typically unknown to the user, and this introduces a degree of freedom in the EKF design that can be difficult to resolve. Furthermore, the EKF produces time histories for each of the estimated parameters, which is very useful if the time-varying nature of the parameters is of interest. However, in most aircraft parameter estimation situations, a single numerical value for each parameter is desired, and these must be derived from the EKF time histories.

This Note presents results from the application of the EKF to the estimation of aerodynamic coefficients for both NASA's X-31 drop model and the high-angle-of-attack research vehicle (HARV) from flight test data, which is part of an ongoing effort to develop systematic procedures for the design of EKFs for parameter estimation. The assumption of a fictitious noise process, or pseudonoise, driving the parameter model is shown to improve the EKF parameter estimates for the HARV. In addition, a residual correlation method[6] (RCM), originally derived for the linear Kalman filter,[4] was used to determine the appropriate process noise intensity and measurement noise covariance matrices for this case.

The Note is organized as follows. The next two sections introduce the aircraft dynamic equations and the EKF parameter estimator. Then, the parameter estimation results for the X-31 drop model and the HARV are presented and discussed. Conclusions follow.

Aircraft Dynamic Equations

The equations of motion for a rigid aircraft can be expressed in the general state-space form[1]

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}[\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{\zeta}, t] + \boldsymbol{w}(t) \tag{1}$$

where the state vector $\boldsymbol{x}$ usually consists of the linear velocity components along the body axes u, v, and w; the angular velocity components about the body axes p, q, and r; and two Euler angles (usually pitch and roll, φ and θ) that describe the orientation of the body axes with respect to a fixed frame of reference. See, for instance, Ref. 1 for a detailed version of Eq. (1). Note that among the terms in Eq. (1) are the aerodynamic forces along the body axes X, Y, and Z and the aerodynamic moments about the body axes L, M, and N, which depend on the aerodynamic coefficients of interest. The vector $\boldsymbol{\zeta}$ contains these parameters. The control input $\boldsymbol{u}$ generally includes the control surface deflections and possibly some thrust-related variables. The vector noise process $\boldsymbol{w}$, which is assumed white with intensity $Q(t)$, represents unknown random perturbation inputs driving the plant (process noise), e.g., perturbation forces and moments arising from atmospheric turbulence.

The output equation representing the available discrete-time measurements is

$$z(k) = \boldsymbol{h}[\boldsymbol{x}(t_k), \boldsymbol{u}(t_k), \zeta, t_k] + \boldsymbol{v}(k) \tag{2}$$

where the output vector z typically includes some subset of the following quantities: the total airspeed $V\,[=\sqrt{(u^2+v^2+w^2)}]$, the angle of attack $\alpha\,[=\tan^{-1}(w/u)]$, the angle of sideslip $\beta\,[=\sin^{-1}(v/V)]$, the angular velocity components, the Euler

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